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A RAPID THERMOMETER FOR MEASUREMENT IN TURBULENT FLOW

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Abstract- A description is given of a thermometer system capable of measuring temperature fluctuations of less than 0.1 K with a frequency of 5000 Hz in air flows. The construction of the sensor, a platinum coated quartz wire, and of the electronic circuitry are described. A theory of the thermal behaviour of the sensor is given. Its response time was measured. The heat-transfer coefficient deduced from these measurements differed markedly from that derived from the *Nu-Re* relation of Collis and Williams [3].

NOMENCLATURE

a,	thermal diffusivity [m^2/s];
c,	specific heat [J/kg·K];
C,	capacitance [F];
Ja,	$= \alpha/(\omega\lambda\rho c)^{\frac{1}{2}}$ Jacob number [dimensionless];
Nu,	Nusselt number [dimensionless];
r,	radius of the cylinder [m];
R,	resistance [Ω];
Re,	Reynolds number [dimensionless];
t,	time [s];
T,	temperature [K, °C];
T_e,	temperature of the environment [K, °C];
U,	air velocity [m/s].

Greek symbols

α,	heat-transfer coefficient [W/m^2·K];
β,	$= (\omega/a)^{\frac{1}{2}}$ [m^{-1}];
η, η',	temperature amplitude ratios [dimensionless];
λ,	thermal conductivity of the material of the core [W/m·K];
λ_a,	thermal conductivity of air [W/m·K];
μ, μ',	transfer functions [dimensionless];
ρ,	density [kg/m^3];
τ,	time constant [s];
τ_{ext},	external time constant [s];
τ_{int},	internal time constant [s];
ϕ, ψ,	phase shifts [rad];
ω,	circular frequency [rad/s].

Mathematical functions

$J_0(r)$,	Bessel function, first kind, order zero;
$ber_0(r), bei_0(r)$,	Bessel–Kelvin functions of order zero defined by the relation $J_0(i^{\frac{3}{2}} r) = ber_0(r) + i\, bei_0(r)$;
$\Phi_R(r)$,	$= ber_0(r) + Ja^{-1} ber'_0(r)$;
$\Phi_I(r)$,	$= bei_0(r) + Ja^{-1} bei'_0(r)$;
$ber'_0(r), bei'_0(r)$,	first derivatives of the Bessel–Kelvin functions with respect to their argument.

1. INTRODUCTION

IN A DIRECT determination of the turbulent heat flux in a turbulent boundary layer in air it is necessary to measure the local temperature and velocity fluctuations simultaneously.

In this article a description is given of an instrument capable of measuring the temperature fluctuations, based on the principle of resistance thermometry. It has a sensitivity of better than 0.1 K and a frequency range up to about 4 kHz.

First a short description is given of the construction and the thermal behaviour of the sensor. Secondly, the electronic circuits used for the current supply and the compensation of the thermal inertia of the sensor are described.

Finally some results are given of measurements of the thermal response of this sensor.

2. CONSTRUCTION OF THE SENSOR

The sensor used here consists of a pyrex or silica cylinder with a diameter of 2–10 µm and a length of 1–2 mm coated with a thin layer of pure platinum with a thickness of about 0.1 µm. This platinum layer acts as a resistance thermometer. The resistance of such a layer is at room temperature in the order of 200–2000 Ω.

The conductive layer was obtained by a diode DC sputtering process in argon at a pressure of 4 Pa. All layers were annealed in a pure hydrogen atmosphere at a temperature of 600–750 K, to remove internal stresses and to reduce the content of platinum oxides (mainly PtO_2). In this way we obtained stable layers with a good adherence to glass and a temperature coefficient of resistance of about 2×10^{-3} K^{-1}.

To illustrate the stability of these layers, we mention that layers made in 1968 kept their original resistance values under laboratory conditions up to now (May 1975) within 1.5%.

We investigated also some other coating techniques such as the deposition of noble metals from liquid Pt and Au preparations and the evaporation of gold in vacuum. However, these processes gave results which were inferior to the sputtering technique.

The platinum coated wire is cemented with silver paint (Degussa Leitsilber No. 200) to the prongs (household steel needles) which are fixed in a two-hole alumina tube by means of an araldite resin. The prongs are bent in such a way that the disturbance of the flow due to the presence of the prongs and the ceramic body is minimized. For the same reason and to diminish the integration effect of the sensor the coated wire is partially coppered (electrolytically) to reduce the sensitive length of the sensor. This is necessary to measure the effect of eddies with a length scale of the same magnitude as the length of the wire (in situations where such small eddies contribute to the heat flux).

3. THE THERMAL BEHAVIOUR OF THE SENSOR

We can consider the sensor as a long cylinder with an electrically insulating core which is coated with an electrically conducting layer exposed to the temperature variations of its surroundings. When we neglect the axial heat conduction to the prongs we have to consider two coupled heat-transfer processes.

(a) The convective transport of heat to the surface of the cylinder. This transport is described by a heat transfer coefficient which is assumed to be constant in the temperature range under consideration.

(b) The conductive transport of heat from the surface via the conductive layer to the interior of the cylinder, which in fact is a two-material laminated cylinder. Fortunately, due to the chosen dimensions and the materials used, we can neglect the presence of the thin platinum layer and consider the sensor as a homogeneous cylinder whose surface temperature is measured.

An extensive study of this subject, also for more complicated cylindrical structures has been made by Lowell and Patton [1].

Here we shall restrict ourselves to the results of most importance for our purpose.

Each heat-transfer process has its own time scale called respectively the external and the internal time constant. Both are responsible for modification of the incoming temperature variation in amplitude and phase.

The external time constant, τ_{ext}, characterizes the system when the cylinder responds as a whole to the external variation; it is given by

$$\tau_{ext} = \rho c r/2\alpha \tag{1}$$

where ρ and c refer to the core material.

In equation (1) we have neglected the influence of the heat production due to the measuring current. To prevent the sensor from acting as an anemometer this current is kept as low as possible. The corresponding error in τ_{ext} is less than 0.05%.

The internal time constant represents the time necessary to let the average temperature of the cylinder attain $(1-e^{-1})$ times its asymptotic change after imposing a stepwise change on the surface temperature. The averaging is done over the cross-section of the cylinder. We have

$$\tau_{int} = 0.111\, r^2/a \tag{2}$$

where a is the thermal diffusivity of the core material.

Examples of time constants under conditions of interest in our research are given in Table 1.

Table 1. Examples of time constants for different diameters, wind velocities and materials

	τ_{ext}[s]		τ_{int}[s]	
	U			
r	1 m/s	16 m/s	Pyrex	Platinum
1 μm	2.1×10^{-4}	9.4×10^{-5}	2.4×10^{-7}	4.7×10^{-9}
5 μm	6.8×10^{-3}	2.6×10^{-3}	6.1×10^{-6}	1.2×10^{-7}

From this table we conclude:

(a) $\tau_{ext} \gg \tau_{int}$; in the most unfavourable case the ratio is still a factor 800.

(b) Both time constants depend strongly on the diameter of the cylinder.

(c) A comparison of the values of τ_{int} for a pyrex cylinder and a platinum cylinder with the same diameter indicates that for our sensor the transfer of heat inside the cylinder is mainly determined by the pyrex core. This confirms the earlier statement that we can neglect the influence of the platinum layer on the thermal behaviour of the sensor.

Combining these facts we see that τ_{ext} plays a dominant role and that it is necessary to aim at diameters as small as possible to minimize the influence of the corresponding thermal inertia.

When we consider a harmonic variation of the temperature of the environment, $T_e \cos \omega t$, then the ratio between the surface temperature $T(r)$ and the amplitude of the imposed temperature wave T_e is given by the following expression [1] (cf. Nomenclature):

$$\eta = \frac{T(r)}{T_e} = \left[\frac{ber_0^2(\beta r) + bei_0^2(\beta r)}{\Phi_R^2(r) + \Phi_I^2(r)}\right]^{\frac{1}{2}} \cos(\omega t - \psi) \tag{3a}$$

with

$$\psi = \arctan\left[\frac{\Phi_I(r) ber_0(\beta r) - \Phi_R(r) bei_0(\beta r)}{\Phi_R(r) ber_0(\beta r) + \Phi_I(r) bei(\beta r)}\right]. \tag{3b}$$

For sufficiently small values for β and Ja these expressions can be simplified to

$$\eta' = \frac{1}{[1 + (\omega\tau_{ext})^2]^{\frac{1}{2}}} \cos(\omega t - \phi) \tag{4a}$$

with

$$\phi = \arctan \omega\tau. \tag{4b}$$

This is the well-known solution for a first order system, which gives the response of a cylinder with a uniform temperature ($\tau_{int} = 0$). To give an illustration of equations (3a) and (3b) in our situation we calculated this response for pyrex cylinders with the same diameters as used in Table 1 and two different values of the heat-transfer coefficient (2000 and 7000 $W \cdot m^{-2} \cdot K^{-1}$).

The results are shown in Fig. 1. For the sake of completeness the phase and amplitude response of the temperature averaged over the cross-section of the cylinder are also given ($\bar{\eta}$ and $\bar{\psi}$).

For the considered frequency range we see from this figure that:

(a) The amplitude response shows in all cases an analogous behaviour as that of a cylinder with a uniform temperature. Differences between the amplitude ratios calculated from equations (3) and (4) are less than 0.1%.

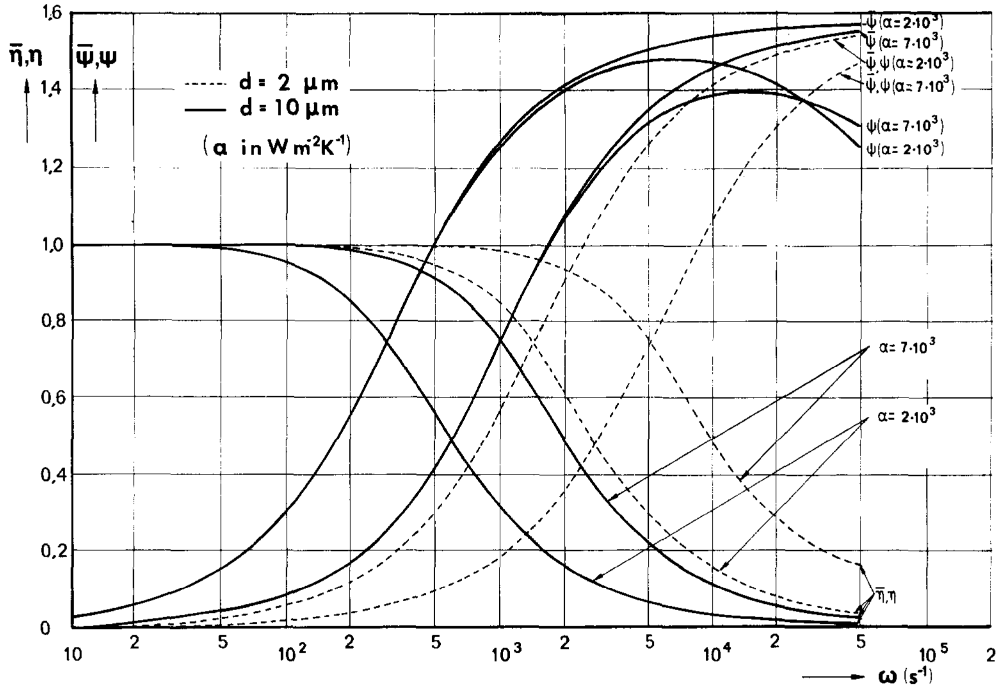

FIG. 1. Amplitude and phase response of pyrex cylinders with diameters of 2 μm and 10 μm.

(b) For the cylinder with the diameter of 10 μm the phase response according to equation (3) shows a maximum due to the influence of τ_{int}, whereas the phase shift according to equation (4) gives a monotoneous increase till $\pi/2$ rad.

From this we conclude that for the materials and dimensions used here, and assuming that it is possible to correct for the influence of τ_{ext}, the limiting factor for the use of this type of sensor is τ_{int}. As τ_{int} is proportional to r^{-2} [cf. equation (2)] the diameter of the cylinder should be taken as small as is permissible from a constructional point of view. It should be noted that the often used relation $\omega\tau = 1$ as a criterion for the influence of a time constant τ is not suited in turbulence research where we need the product of the instantaneous values of two or more fluctuating quantities. It is a far better approach to use $\omega\tau = 0.1$. This leads, for instance, on the basis of equation (4) to an acceptable maximum phase shift of 0.1 rad and amplitude reduction of 0.5%.

4. COMPENSATION NETWORK FOR τ_{ext}

From Section 3 we know that the useful frequency range is mainly restricted by τ_{ext}. If we only consider this parameter the complex transfer function $\mu(\omega)$ for the ratio of the temperature of the sensor T and the instantaneous temperature of its environment T_e' (to be distinguished from the amplitude of the temperature T_e as given in Section 3) is [cf. equation (4)]:

$$\mu(\omega) = \frac{T}{T_e'} = \frac{1}{1 + i\omega\tau_{\text{ext}}} = \frac{1}{\sqrt{[1 + (\omega\tau_{\text{ext}})^2]}} \exp(-i\omega\tau_{\text{ext}}). \tag{5}$$

Thus the influence of the thermal inertia of the sensor can be compensated by a correction network with the transfer function

$$\mu'(\omega) = 1 + i\omega\tau_{\text{corr}}, \text{ with } \tau_{\text{corr}} = \tau_{\text{ext}}. \tag{6}$$

The circuit shown in Fig. 2 has the desired transfer function. The time constant of this circuit τ_{corr} is formed by R_1 and C_1 weighted by the ratio R_0/R_v so that the required result is achieved by choosing.

$$R_v = R_1 C_1 R_0/\tau_{\text{ext}}. \tag{7}$$

The first operational amplifier used in this circuit only serves as an insulating amplifier. In this way we obtain a frequency independent loading of the current supply circuit.

The effect of the differentiating part of this network is to give an output which is proportional to the frequency of the input. To prevent the undesired contribution of noise with frequencies higher than the range of interest we restricted the differentiating action till a frequency of 4000 Hz. Above this frequency the amplification decreases with 6 dB per octave.

5. ELECTRIC SUPPLY

We did not use a Wheatstone bridge, as is normally applied in accurate resistance thermometry, because the non-linear relationship between an impedance change in one of the branches of the bridge and the output prevents an adequate compensation for the spurious tensions caused by the presence of stray capacities and inductances over the whole frequency range.

The modern development of solid state technology makes it possible to construct constant current supplies for the required low currents (below 1 mA) with a

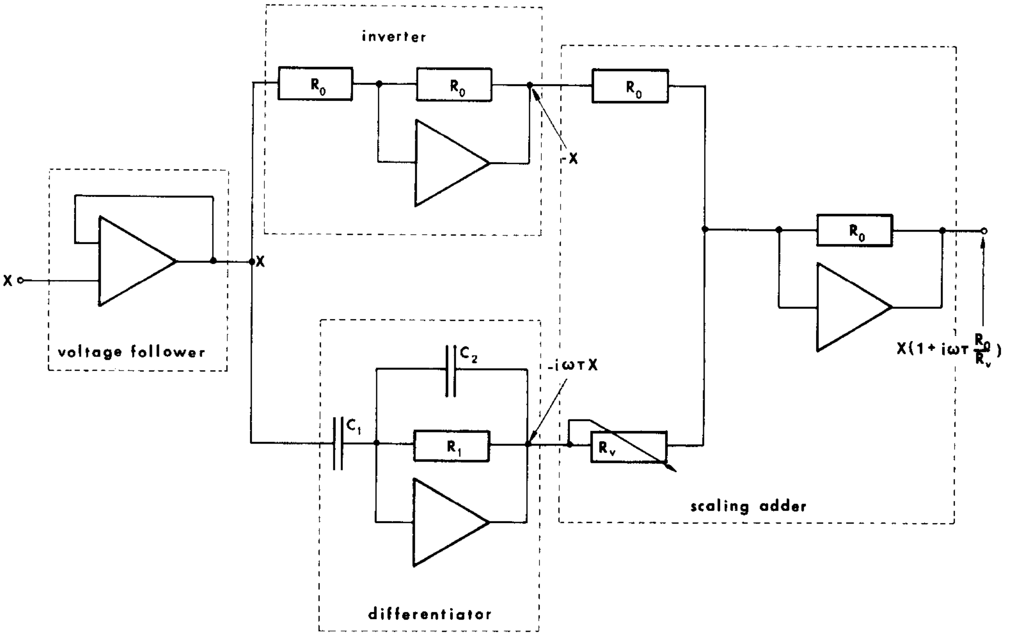

FIG. 2. Compensation network for τ_{ext}.

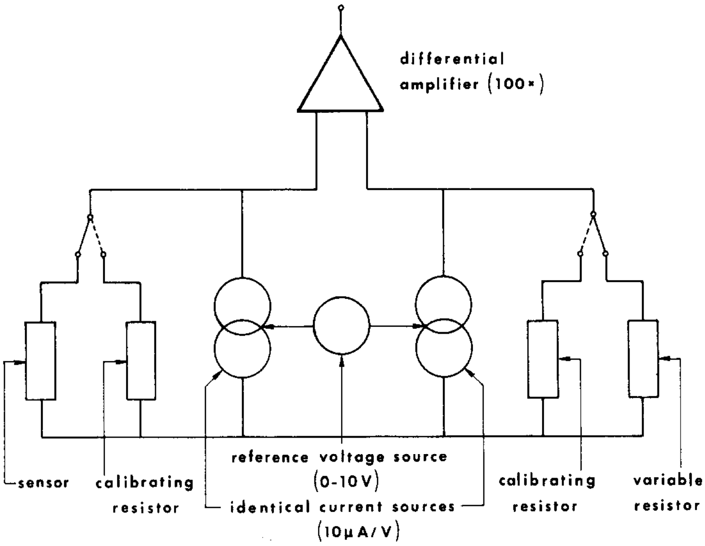

FIG. 3. Constant current supply.

response time small enough to follow the fastest resistance changes of the sensor.

The supply, developed in our laboratory and shown in Fig. 3, consists of two identical current sources which obtain their reference tensions from one constant tension source. One of the current sources feeds the sensor the other a variable resistor which in most cases is adjusted to the expected mean resistance value of the sensor. The potential difference between the two resistors is fed into an amplifier with a fixed gain of 100 to bring the signal to an adequate level for further processing.

The current supply has a range of 0–100 µA. The noise figure in the frequency range from 0.06 to 6 kHz is 0.75 mV, referred to output. The temperature drift could be kept very low due to the symmetry of this network. When, for instance, we use a sensor with a resistance of 1000 Ω and a temperature coefficient of resistance of $2 \times 10^{-3}\,K^{-1}$ we can detect temperature variations with an amplitude of less than 0.075 K. A change of 1 K in the ambient temperature corresponds with an apparent change of the order of 7×10^{-3} K in the temperature of the sensor, due to the temperature drift of the circuit.

When the length of the cables between the current supply and the sensor is less than 10 m the time constant is below 5 µs. Using longer cables two additional capacitors have to be added to prevent oscillating. In the latter case the time constant is increased to 20 µs. In the former case the supply can be used till frequencies beyond 4 kHz, in the latter case till about 1 kHz.

6. EXPERIMENTAL DETERMINATION OF τ_{ext}

For the determination of the time constant τ_{ext} of the sensor as a function of the wind speed U, a somewhat modified version of the method of Kunstman and Kivnick [2] was used. The sensor was heated by a direct current plus a low frequency amplitude modulated high frequency current. Since the time constants of the wire are too large to permit the sensor to follow the high frequency oscillations this carrier wave gives rise to a constant heating power. The desired value of τ_{ext} is found from the circular frequency, ω, of the low frequency modulating signal that gives a phase shift of $\pi/2$ rad between the current through the sensor and the potential difference over the sensor, according to equation (4b).

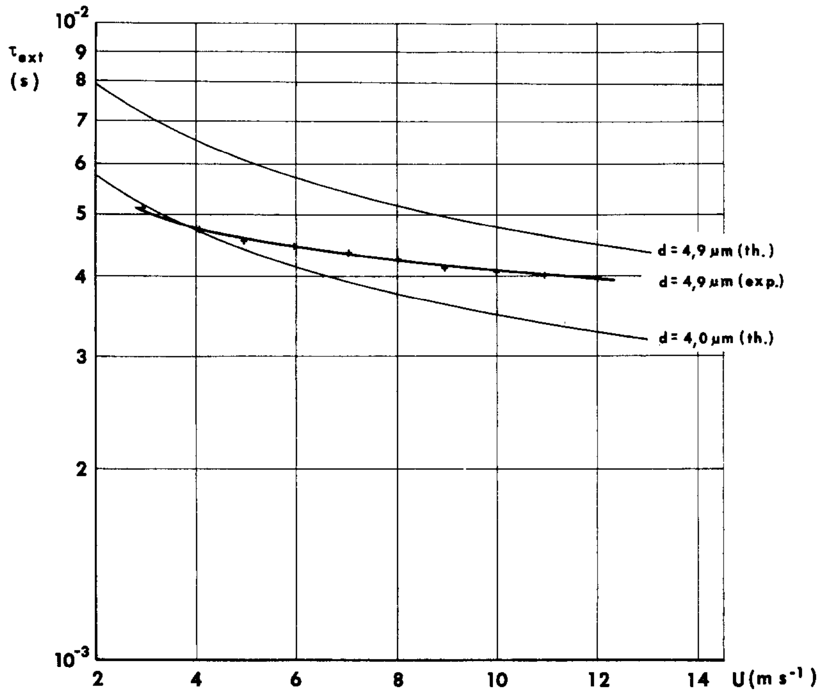

FIG. 4. Comparison between τ_{ext}–U relations found from experiments and from the Collis and Williams relation.

It is assumed that it makes no difference in the heat transfer when the temperature difference between the sensor and its environment is caused by the temperature change of the environment or by the heating of the sensor by means of an electric current.

It is easy to derive τ_{ext} as a function of U from one of the well known semi-empirical *Nu–Re* relations holding for a cylinder under conditions of stationary forced convection.

From equation (1) it follows that

$$\tau_{ext} = \frac{\rho c r^2}{Nu\lambda_a}. \tag{8}$$

In Fig. 4 we compared the results according to both methods using the *Nu–Re* relation of Collis and Williams [3] for a pyrex sensor with a diameter of 4.9 μm. A second curve for a diameter of 4.0 μm shows the influence of a possible error in the determination of the diameter of the cylinder. The estimated accuracy of our determination of the diameter of the sensors was about 5%.

From this figure we see that in a qualitative sense there is a fair agreement between both methods. However, the slope of the curves for d = 4.9 µm and 4.0 µm show a remarkable difference with that of the curve derived from the *Nu–Re* relation. This discrepancy cannot be ascribed to the influence of the heating current or to the heat conduction to the supports, since both effects are negligible here. The main difference between both methods is that in our measurements the dynamical behaviour of the thermal boundary layer can play a role whereas the other experiments are carried out under stationary conditions.

7. CONCLUSIONS

The measuring system described above fulfils the requirements mentioned in the Introduction.

The major advantages of the use of this type of sensor, as compared with a more conventional type consisting of a homogeneous metal cylinder, lies not in its thermal behaviour, because τ_{int} of the latter is always smaller than that of our sensor. The advantages are its superior strength, the lower heat conduction to the supports and the higher electrical impedance. The latter property results in a better signal to noise ratio due to the better impedance matching with the electronic circuit.

It remains necessary to investigate further the dependency of τ_{ext} on the wind velocity.

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UN THERMOMETRE RAPIDE POUR MESURE DANS DES ECOULEMENTS TURBULENTS

Résumé—Il est présenté un système avec lequel on peut mesurer les fluctuations de température inférieure à 0,1K, à une fréquence de 5000 Hz, dans les écoulements d'air. On décrit la construction du capteur (un fil de quartz couvert d'une couche mince en platine) et celle des circuits électroniques. La théorie du comportement thermique du capteur est présentée. La mesure du temps de réponse a conduit à une différence remarquable entre le coefficient de transfert de chaleur calculé à partir de ces mesures et celui dérivé de la relation *Nu–Re* de Collis et Williams.

EIN SCHNELLANSPRECHENDES THERMOMETER FÜR MESSUNGEN IN TURBULENTER STRÖMUNG

Zusammenfassung—Es wird eine Beschreibung gegeben eines Thermometers das Temperaturschwankungen von weniger als 0,1 K mit einer Frequenz von 5000 Hz in Luftströmungen messen kann. Die Konstruktion des Fühlers, eines platinbeschichteten Quarzdrahtes und des elektronischen Messkreises sind beschrieben. Eine Theorie des thermischen Verhaltens des Fühlers wird angegeben. Die Ansprechzeit wurde gemessen. Die aus den Messungen sich ergebenden Wärmeübergangskoeffizienten unterscheiden sich sehr wesentlich von den nach Nusselt—Beziehungen von Collis und Williams abgeleiteten.

МАЛОИНЕРЦИОННЫЙ ТЕРМОМЕТР ДЛЯ ИЗМЕРЕНИЯ ФЛУКТУАЦИЙ ТЕМПЕРАТУРЫ В ТУРБУЛЕНТНОМ ПОТОКЕ

Аннотация — Рассматривается термометрическая система для измерения температурных флуктуаций менее 0.1°K с частотой до 5000 гц в потоках воздуха. Описывается конструкция датчика, представляющего собой платиновую проволочку с кварцевым покрытием, а также электронная схема. Даётся теоретическое обоснование термического поведения датчика, измеряется его временная характеристика. Значения коэффициента теплообмена, полученные с помощью этих измерений, значительно отличаются от значений, полученных по соотношению *Nu–Re* Коллизом и Вильямсом.